Geometer's Sketchpad Assignment

Using *Geometer's Sketchpad*
To Explore the Centers, Circles, Points, and Lines
Associated with a Triangle

You will be constructing and exploring, special points associated with triangles. *Construct all figures to have the greatest generality possible*. Label the figures thoughtfully and write observations for each question. The figures and observations to each question are to be saved onto ONE Geometer's Sketchpad file, using tabs.

Note: when asked for an explanation, a proof is required. This proof can be something you create, or one that you research. In the latter case, an effort must be made to present the proof in your own words, rather than simply copying it from the source. (If using a source, it *must* be cited.)

1. Construct a *general triangle* and the midpoint of each side. Connect each midpoint to the opposite vertex to form the three **medians** of the triangle.
   Construct the point at which any two of the three medians meet; this point is the **centroid,** or center of gravity, of the triangle. The centroid is a **point of concurrence** of the three medians. (Note: The software does not recognize a point of concurrence for three straight objects.) Explain why the centroid is called the center of gravity of the triangle.

2. Open a new sketch. Construct a general triangle as before. Construct the perpendicular bisectors of the three sides. They meet at the **circumcenter,** the center of the circle passing through the three vertices of the triangle. What do you observe about the circumcenter? Use the circumcenter to construct the associated **circumcircle.** Explain why this works.

3. Open a new sketch. Construct a general triangle as before. Construct the angle bisectors of the three interior angles. They meet at the **incenter,** the center of the inscribed circle. Construct the associated **incircle.** Explain why the incircle remains inscribed inside the triangle.

4. Open a new sketch. Construct three points. Connect the points with lines (rather than segments) to form an **extended triangle.** Construct the three **altitudes** of the triangle, the perpendiculars from the vertices to the extended opposite sides. The altitudes meet at the **orthocenter,** which together with the three vertices form the triangle's **orthocentric set.** Explain why the altitudes are concurrent. Save this sketch for use in Activities 5 and 6 below.

5. Copy the sketch in #4 on a new page. Three of the four centers defined and explored so far are collinear. The line determined by these three centers is the **Euler line** of a triangle. Construct the Euler line of an extended triangle. Which three centers lie on the Euler line? Indicate this on your sketch. Look up a proof of the Euler line and give a brief summary of the proof.

6. Copy the sketch in #4 on another new page. Construct the midpoints of the three sides of a triangle and the three midpoints of the segments from the orthocenter to the three vertices. Construct a circle that contains these six midpoints. This is the triangle's **nine-point circle.** Which other three points does the nine-point circle contain? Explain how to determine the center of the nine point circle.

7. Construct an extended triangle. Construct a point on each of the three sides. For each vertex of the triangle, construct a circle passing through the vertex and the points on the two adjacent sides. Use *Sketchpad* to find that the three circles contain a common point. This is a **Miquel point** of the triangle. Move the vertices of the triangle and move the points on the extended sides along the extended sides. What do you observe? Find a proof that explains what you're observing.

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