

1. Define what it means for an argument to be deductively valid.

A deductively valid argument is one in which IF the premises are true, THEN the conclusion HAS TO BE true. (Or equivalent formulations)

2. Construct a truth table to show whether or not these two formulas are logically equivalent.

$$\sim P \vee R \qquad P \supset R$$

P	$\sim P$	R	$\sim P \vee R$	$P \supset R$
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T

3. (a) Explain the general process used in Indirect Proof and (b) use it to solve this problem:

1. $D \supset \sim(A \vee B)$
2. $\sim C \supset D$ TO PROVE: $A \supset C$
3. $\sim(A \supset C)$ AP The negation of the conclusion is assumed as an additional premise
4. $\sim(\sim A \vee C)$ 3, by Implication
5. $A \ \& \ \sim C$ 4, by De Morgan's Laws
6. $\sim C$ 5, by Simplification
7. D 6, 2 by Modus Ponens
8. $\sim(A \vee B)$ 7, 1 by Modus Ponens
9. $\sim A \ \& \ \sim B$ 8, by De Morgan's Laws
10. $\sim A$ 9, by Simplification
11. A 5, by Simplification
12. $A \ \& \ \sim A$ 10,11 Conjunction
13. $A \supset C$ 3 to 12, Indirect Proof

Explanation: the negation of the conclusion leads through a series of deductively valid steps to a contradiction. That proves that the negation of the conclusion has to be false in relation to the initial premises, and thus the conclusion must be true.

[Indirect Proofs are a good way to test for a number of proof techniques in one problem besides being a hugely important proof technique in its own right.]

4. Translate the following statement into symbolic notation:

(a) The Astros will win the pennant only if their bullpen improves.

W = Astros will win the pennant, I = their bull pen improves, $W \supset I$

[The "only if" is actually equivalent to IF...THEN symbolized by the \supset]

5. Translate the following statement into symbolic notation:

(b) The hurricane will come ashore at Galveston unless it changes course soon.

G = The hurricane will come ashore at Galveston, C = it changes course soon

Either $\sim C \supset G$ or $C \vee G$ [Each formula is equivalent to the other.]

6. Give the forms of these basic logical equivalences:

(a) De Morgan's Laws $\sim(A \& B) = \sim A \vee \sim B$ and $\sim(A \vee B) = \sim A \& \sim B$

(b) Contraposition $A \supset B = \sim B \supset \sim A$

7. Translate the following into symbolic notation:

(a) a Universal Affirmative statement such as "All sailors wear beards" = $(x) (Sx \supset Bx)$

(b) a Universal Negative statement such as "No sailors wear beards" = $(x) (Sx \supset \sim Bx)$

8. Translate the following into symbolic notation:

(a) a Particular Affirmative statement such as "Some sailors wear beards" = $(\exists x) (Sx \& Bx)$

(b) A Particular Negative statement such as "Some sailors do not wear beards" = $(\exists x) (Sx \& \sim Bx)$

9. Give the simple logical forms for refutation and confirmation during the process of hypothesis testing and explain why refutation in this form is logically stronger than confirmation:

H = the hypothesis being tested

P = a predicted observable result

Confirmation

1. $H \supset P$

2. \underline{P}

3. H

Refutation

4. $H \supset P$

5. $\underline{\sim P}$

6. $\sim H$

Refutation is stronger than confirmation because refutation employs Modus Tollens, a deductively valid argument form, whereas confirmation employs the Affirming the Consequent argument form which is not deductively valid.

10. Symbolize the premises and the conclusion in the following argument and then prove how the conclusion follows from the premises by using the rules of inference involving quantifiers.

No ravens are pink, but some flamingos are pink, so therefore some flamingos are not ravens.

1. $(x) (Rx \supset \sim Px)$ Premise [No ravens are pink]
2. $(\exists x) (Fx \& Px)$ Premise [Some flamingos are pink]
3. $Fx \& Px$ 2, Existential Instantiation
4. $Rx \supset \sim Px$ 1, Universal Instantiation
5. Px 3, Simplification
6. $\sim Rx$ 4,5 Modus Tollens
7. Fx 3, Simplification
8. $Fx \& \sim Rx$ 6,7 Conjunction
9. $(\exists x) (Fx \& \sim Rx)$ 8, Existential Generalization [Some flamingos are not ravens]