GeoGebra Project
To Explore the Centers, Circles, Points, and Lines Associated with a Triangle

For this GeoGebra assignment, you will be constructing, exploring, and printing various related to triangles *Construct all figures to have the greatest generality possible*. Label the figures thoughtfully and write observations for each activity.

Medians and the Centroid

1. Construct a *general triangle* ABC. Construct the midpoint of each side and connect it to the opposite vertex to form the three **medians** of the triangle. Label the midpoints D, E, and F in such a way that D lies on the side opposite A, E lies on the side opposite B, and F lies on the side opposite C. What do the medians have in common? Use the drag test to verify this is true.

2. The medians are concurrent. Mark this point of intersection G. Measure AG and GD and then calculate AG/GD. Make an observation about the ratio. Do the same for the segments associated with the other vertices and midpoints. Make an observation about the ratios

3. The centroid is a **point of concurrence** of the three medians. Explain why the centroid is called the center of gravity of the triangle.

4. You should have discovered that the three medians are concurrent. The other observations you've noted should lead you to make a conjecture. Let's call it the Median Concurrence Theorem. Write a statement of the theorem. A proof is NOT required.

5. The medians subdivide the triangle ABC into six smaller triangles. Determine the shape of triangle ABC for which the subtriangles are congruent.

Altitudes and the Orthocenter

6. Open a new sketch. Construct a general triangle ABC as before. Construct the altitudes of its sides. Verify that they are concurrent and meet at the **orthocenter.**
7. Construct another triangle. Mark the orthocenter of this triangle and label it H. Add the centroid, G, to your sketch and again move the vertices and observe H and G. Notice that while G always appears to remain in the interior of the triangle, this is not true for the orthocenter.
8. Determine by experimentation that shape of triangles for which the orthocenter is outside the triangle. Find a shape for the triangle so that the orthocenter is equal to one of the vertices.
9. Determine by experimentation whether or not it is possible for the centroid and the orthocenter to be the same point. If it is possible, for which triangles does this happen?

Perpendicular bisectors and the Circumcenter

10. Open a new sketch. Construct a general triangle ABC as before. Construct the perpendicular bisectors if its sides. They meet at the **circumcenter,** label it O.
11. Move one vertex and observe how the circumcenter changes. Note what happens when on vertex crosses over the line determined by the other two vertices. Find triangles that show that the circumcenter may be inside, on, or outside the triangle, depending on the shape of your triangle. Make a note of your findings.
12. Measure OA, OB, and OC and make an observation about them.
13. Prove that the circumcenter is equidistant from the vertices of the triangle.
14. Determine by experimentation whether or not it is possible for the circumcenter and centroid to be the same point. If it is possible, for which triangles does this happen?

Angle Bisectors and the Incenter

15. Open a new sketch. Construct a general triangle as before. Construct the angle bisectors of the three interior angles. They meet at the **incenter,** the center of the inscribed circle. Label this point I.
16. Use I to construct the associated **incircle.** Explain how this is done.

The Euler Line

17. Three of the four centers defined and explored so far are collinear. The line determined by these three centers is the **Euler line** of a triangle. Construct the Euler line of a triangle. Which three centers lie on the Euler line?
18. Measure distances HG and GO. Calculate HG/GO. Leave the calculations visible on your screen as you change the shape of your triangle. What happens to the ratio and the triangle changes?
19. Use the results of your observations to 17 and 18 to state the **Euler Line Theorem**.

Exercises

1. Prove that a triangle is equilateral iff its centroid and circumcenter are the same point.
2. Prove that a triangle is isosceles iff two medians are congruent.
3. Prove that a triangle is isosceles iff two altitudes are congruent.